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A BRIEF ACCOUNT OF THE ESSENTIAL FEATURES OF GRASSMANN'S EXTENSIVE ALGEBRA.*

[Given by the author in Grunert's Archiv, Vol. VI, 1845.]

TRANSLATED BY PROF. W. W. BEMAN, ANN ARBOR, MICH.

I. Aim of the Extensive Algebra as Such.

1. *My Extensive Algebra forms the abstract foundation of Geometry,* i. e., *it is the pure mathematical science divested of all spatial considerations, whose special application to space is Geometry.*

Geometry, inasmuch as it rests upon something given in nature, viz., space, is not a branch of pure mathematics, but an application of the same to nature; yet not a mere application of algebra, even when the algebraic quantity, as in the theory of functions, is regarded as varying continuously; for algebra lacks the notion of different dimensions, which is characteristic of geometry. Hence we need a branch of mathematics which incorporates with the notion of the continuously varying quantity the notion of differences (Verschiedenheiten) corresponding to the dimensions of space. This branch is my extensive algebra.

2. *The theorems of the extensive algebra are not mere translations of geometrical theorems into abstract language; they have a far more general significance, for while geometry is restricted to the three dimensions of space, the abstract science is free from these restrictions.*

In geometry, lines may be generated by the movement of points, surfaces by the movement of lines, volumes by the movement of surfaces, but further geometry cannot go. On the contrary, if we conceive abstract notions independent of space to be introduced in the place of the point and movement, these restrictions disappear.

3. *Hence it follows that the theorems of geometry have a tendency toward generality, which finds no satisfaction in geometry, on account of its restriction to three dimensions, but does find complete satisfaction for the first time in the extensive algebra.*

A couple of examples will make this plain. (1) Two right lines in the same plane intersect in one point, likewise a plane and a right line, two planes intersect in one right line, provided that the right lines, or the plane and the right line, or the two planes, do not coincide, and the intersections at infinity are included. If the point, the right line, the plane, and the solid, be regarded as fields (Gebiete) of the first, second, third, and fourth orders (Stufe) respectively, then in this is suggested the general theorem, that if a field of the ath order and one of the bth intersect in a field of the cth order, but not in a field of lower order, they will have a field of the $(a+b-c)$th order in common. Geometry, however, can illustrate this theorem only when c is equal to, or less than, 4. (2) The area of a triangle is half that of a parallelogram whose sides are equal in length and parallel, to two sides of the triangle; the volume of a tetraedron is one sixth that of the parallelepiped whose edges are equal in length and parallel, to three edges of the tetraedron meeting in a common point. In this is evidently suggested the theorem, that the space included within n points taken together in a field of the nth order (and not in a field of lower order) is $\frac{1}{1.2.3\ldots(n-1)}$th of the space enclosed by a figure (Gebilde) whose bounding lines (edges) are equal in length and parallel, to the right lines joining one of the n points to all the rest. But even here this theorem does not appear in all its generality. On the contrary, in the extensive algebra—in these two cases, and in all others — the theorems hold true in their most general form. Thus everywhere geometry takes the first step toward generality, but without being able to reach it, strikes against the barriers set up by the idea of space, which barriers only the abstract science of extensive algebra is capable of breaking down.

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*Prof. Clifford's translation of "Ausdehnungslehre". Tr.

4. *The figure (Gebilde) in the extensive algebra which corresponds to the line is the aggregate of the elements resulting from a continuous variation of the state of the original element.*

The line may be regarded as the aggregate of the points resulting from a continuous change of position of a point. If we substitute here for the p't, more generally, any object susceptible of a continuous variation of any state in which it is, and then divest it of every other property and every singularity of its state, and call the object, thus divested of every other property, the element, we arrive at the idea set forth.

5. *If now the element continuously varies its state in the same way, so that after passing from an element* a *of the figure (Gebilde) to a second element* b *of the same by one such variation, by a second we pass from* B *to an element* c *of the same figure, we obtain the figure corresponding to the* RIGHT *line, viz., the field of the second order.*

The right line is generated by the point continually varying its position in the same direction; if we substitute for direction mode of variation, we arrive at the idea set forth.*

[To be continued.]

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*If the right line and the figure corresponding to it are to be infinite both ways, the point (the element) must vary its position in the opposite direction (its state in the opposite manner) also, a thought which for simplicity we have neglected here.